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A STUDY OF THE EFFECTS OF VERTICAL TAIL AREA AND DIHEDRAL ON THE LATERAL MANEUVERABILITY OF AN AIRPLANE

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A STUDY OF THE EFFECTS OF VERTICAL TAIL AREA AND DIHEDRAL ON THE LATERAL MANEUVERABILITY OF AN AIRPLANE

By Leo F. Fehlner

SUMMARY

A theoretical investigation has been made of the influence of changes in vertical tail area and dihedral angle on the lateral flying characteristics of an airplane. The lateral motions were computed for nine different combinations of vertical tail area and dihedral, covering the range ox" parameters used in the. design of typical airplanes. The motions were initiated by aileron and rudder controls, and. consideration was given to various amounts of yawing moment accompanying aileron control.

Aileron control was shown to produce better lateral maneuvering than rudder control. With aileron control, improvements were accomplished by increases in vertical tail area and decreases in dihedral. The rudder was effective for enforcing sideslip and supplementing the ailerons,

The possibility is advanaed that favorable lateral flying characteristics are associated with very short-period lateral oscillations.

INTRODUCTION

In the design of airplanes, several criterions have been suggested and used for the choice of aerodynamic derivatives to obtain favorable flying characteristics. The results of complying with different existing criterions for lateral chnracteristics, however, are at variance as evidenced by the number of airplanes that show widely different lateral Plying characteristics. The present paper is an endeavor to establish a comprehensive basis for the choice of lateral aerodynamic derivatives, Lateral flying qualities are known to be Largely dependent on vertical teil area and dihedral angle but, for making predictions, the nature and the magnitude of the effects of specific changes remain to be considered. Consideration must also Le given to the lateral eontsols because desirable qualities are, in part at least, dependent on tho available control and the skill with which it must be applied.

In the present study, therefore, the effects of definite changes in vertical tail area and dihedral on the lateral maneuverability of an airplane are investigated; computations are made of the banking, the azimuth, and the sideslipping motions for a hypothetical airplane with three different amounts of vertical tail area and three settings of dihedral angle. The motions were initiated by rudder control alone and by aileron control with various supplementary yawing moments.

The results have been analyzed on the basis of the attainment of optimum lateral flying characteristics with a minimum of effort. With aileron control, for instance,, optimum attributes of the aotions are a linear variation of angle of bank of as large a magnitude as is feasible witn a given amount of control, tne elimination of adverse heading, and a minimum of sideslip. The control moment required to obtain a given angular displacement in a given time was takes as a measure of control fcrcs. The data are presented in a form that should be of aid in making qualitative estimates of the lateral flying characteristics of conventional airplanes.

METHOD

The analysis of the effect of fin area and dihedral on the maneuverability of an airplane was made by a comparative study of the lateral motions resulting from the use of the ailerons and the rudder.

The lateral motions of the airplane may be obtained as solations to the lateral equations of motion. These equations contain terns that are dependent on the parameters: fin area and dihedral. Changes in the parameters effect associated changes in the lateral motions. Motions fur every combination of these parameters may therefore be determined.

The dimensional equations of lateral motion are essentially identical with those used in the past and their development may be obtained from reference 1. The equations are as follows:

$$\left.\begin{aligned}
mk_X^2 \frac{dp}{dt} - pL_p - \beta L_\beta - rL_r &= L \\
mk_Z^2 \frac{dr}{dt} - pN_p - \beta N_\beta - rN_r &= N \\
m V \frac{d\beta}{dt} - mg\phi - \beta Y_\beta + rmV &= Y
\end{aligned}\right\} \qquad (1)$$

where the symbols are standard as listed on the covers of NACA reports. In addition

$$L_p = \frac{\partial L}{\partial p},\ N_\beta = \frac{\partial N}{\partial \beta},\ \text{etc.}$$

$$\text{and} \quad \beta = \frac{v}{V}$$

In order conveniently to use aerodynamic data in standard nondimensional form and simultaneously to maintain the original physical significance of the equations, a consistent nondimensional system was used.

The fundamental units of the nondimensional system may be taken to be

b unit of length

b/V unit of time

$\frac{\rho}{2} S_W b$ unit of mass

If length, time, and mass are expressed in terms of the proper units, there will result nondimensional quantities as follows:

$$\left.\begin{aligned} K &= \frac{k}{b} \\ s &= \frac{t}{b/V} \\ \mu &= \frac{m}{\frac{\rho}{2} S_W b} \end{aligned}\right\} \qquad (2)$$

If the dimensional moment equations are divided through by $S_W \frac{\rho}{2} V^2 b$ and the force equation by $S_W \frac{\rho}{2} V^2$, and if the proper **substitutions** are made according to equations (2), the nondimensional equations of lateral motion are

$$\left.\begin{aligned} &\mu K_X^2 D^2\phi - D\phi C_{l_{D\phi}} - \beta C_{l_\beta} - D\psi C_l \\ &\mu K_Z^2 D^2\psi - D\phi C_{n_{D\phi}} - \beta C_{n_\beta} - D\psi C_{n_{D\psi}} = C_n \\ &\mu\, D\rho \quad - \phi C_L \quad - \beta C_{Y_\beta} + D\psi\mu \quad = C_Y \end{aligned}\right\} \qquad (3)$$

where

$$D = \frac{d}{ds}$$

$$C_{l_{D\phi}} = \frac{\partial C_l}{\partial D\phi}, \quad C_{n_\beta} = \frac{\partial C_n}{\partial \beta}, \quad \text{etc.}$$

It should be noted that

$$D\phi = \frac{d\phi}{dt} \; \frac{dt}{ds} = \frac{pb}{V}$$

Thus the partial derivatives $C_{l_{D\phi}}$, $C_{n_{D\psi}}$, etc. are half the value of the derivatives C_{l_p}, C_{n_r}, etc., where

$$C_{l_p} = \frac{\partial C_l}{\partial \frac{pb}{2V}}, \quad C_{n_r} = \frac{\partial C_n}{\partial \frac{rb}{2V}}, \quad \text{etc.}$$

The terms of the right-hand side of equations (3) represent disturbances applied to the system and may be functions of time or, nondimensionally, functions of the distance traveled. The type of disturbance assumed for all cases in this study is one that is applied at the beginning of the motion and held constant thereafter. Such disturbances are represented by $C_l 1(s)$ and $C_n 1(s)$, where the coefficients C_l and C_n represent the magnitude of the applied couples. Because it can be shown that the results obtained, if the rudder is assumed to apply a pure moment, are altered by a multiplying factor of the order of 0.997 or 1.003 when C_Y is taken into account, C_Y may be assumed to be zero.

Equations (3) can now be solved for the variables ϕ, β, and ψ and the solutions as functions of s are the banking, sideslipping, and azimuth motions, respectively. The unit solutions are obtained by the methods of operational calculus in a manner similar to that outlined in reference 2 and the solutions are of the form

$$A + B(s) + C e^{\lambda_1 s} + E e^{\lambda_2 s} \dots$$

The solutions fur ϕ can also be written

$$\phi = C_l \phi_l + C_n \phi_n \tag{4}$$

where the banking motion ϕ_l is due to a unit rolling disturbance, ϕ_n is due to a unit yawing disturbance, and ϕ is the banking motion resulting from any combination of magnitudes of the two disturbances,

Likewise,

$$\psi = C_l \psi_l + C_n \psi_n \tag{5}$$

and

$$\beta = C_l \, \beta_l + C_n \, \beta_n \tag{6}$$

Equations (4), (5), and (6) express in general terms the solutions to the lateral equations of motion. For convenience, however, each solution may be rewritten in various forms to demonstrate more clearly certain paints. Equation (4), for instance, may be written

$$\frac{\phi}{C_l} = \phi_l + \frac{C_n}{C_l} \phi_n \tag{7}$$

where ϕ/C_l is the motion associated with the application of a unit rolling moment and simultaneously a yawing moment in the ratio C_n/C_l.

Similarly,

$$\frac{\psi}{C_l} = \psi_l + \frac{C_n}{C_l} \psi_n \tag{8}$$

and

$$\frac{\beta}{C_l} = \beta_l + \frac{C_n}{C_l} \beta_n \tag{9}$$

Up to this point, only motions have been considered that have been initiated by predetermined disturbances. It is also possible to study disturbances necessary to perform predetermined motions.

Thus, the motion to be predetermined may be that motion which results in the angle o' bank a at the end of the interval s_o. Such a motion is referred to as a "maneuver" and will be labeled ϕ_{s_o}. Because ϕ_l and ϕ_n are known functions of s, their values at the end of the interval s_o may be determined and called $\phi_{l_{s_o}}$ and $\phi_{n_{s_o}}$, respectively. Then, in order to study control directly, the reciprocal of equation (7) mag be n and at the instant s_o

$$\frac{C_l}{\phi_{s_o}} = \frac{1}{\phi_{l_{s_o}} + \frac{C_n}{C_l}\phi_{n_{s_o}}} \qquad (10)$$

Thus, for any given maneuver and type of control, the effect of changing vertical tail area and dihedral on the magnitude of the necessary control may be determined.

A consideration of optimum conditions for controlled motions indicated that the reaction to control in any case should be in strict accordance with the intended reaction. With respect to the azimuth motion, any change in heading opposite to that intended is undesirable and such a reaction will be termed "adverse."

In order to study the elimination of an adverse heading, consider the hypothetical case in which particular control applications prevent any change of heading. Then, equation (8) may be written

$$C_l \psi_l + C_n \psi_n = 0,$$

or

$$\frac{C_n}{C_l} = -\frac{\psi_l}{\psi_n} \qquad (11)$$

Thus the variation of C_n/C_l required to prevent a change of heading is determined as a function of s. The function $\frac{C_n}{C_l}$ (s), for the cases considered, has positive maximum values which indicate that the resulting heading will always be favorable and not adverse if control is applied initially in the ratio C_n/C_l equal to or greater than that maximum determined by the function,

In order to determine the lateral motions following applied yawing moment alone, equation (4) becomes

$$\phi = C_n \phi_n$$

or

$$\frac{\phi}{C_n} = \phi_n \qquad (12)$$

In like manner, from equation (5)

$$\frac{\psi}{C_n} = \psi_n \qquad (13)$$

and from equation (6)

$$\frac{\beta}{C_n} = \beta_n \qquad (14)$$

ASSUMED AIRPLANE CHARACTERISTICS

In order to obtain solutions for ϕ, β, and ψ that can be plotted for analytic purposes, numerical values of the stability derivatives and other airplane characteristics must be determined. A pursuit-type airplane mas assumed and the weight distribution was taken as the average of 19 modern pursuit airplanes. Although of conventional design and general dimensions, the airplane in detail was assumed to be characterized chiefly by its set of aerodynamic derivatives.

Pertinent numerical data are as follows: the span is assumed to be 32 feet, the aspect ratio, 8; the taper ratio, 2:1; and the sweep angle, 0". The lift coefficient during flight is assumed to be 1. Beferences 3, 4, and 5 were used for determining representative values of the stability derivatives, Those derivatives, which are usually considered essentially independent of changes in fin area and dihedral, are as follows:

$$C_{l_p} = -0.5$$

$$C_{l_r} = 0.344$$

$$C_{n_p} = -0.0644$$

The principal effect of varying dihedral is to change the amount of rolling moment due to sideslip. Thus

Γ, deg	C_{l_β}
0	0
5	-.07
10	-.14

where Γ is the effective dihedral angle.

Changes in fin area influence the values of certain stability derivatives, namely, yawing moment and side force due to sideslip, and yawing moment due to yawing. All other effects are considered small. Thus

$\frac{S_f}{S_w}$	C_{n_β}	C_{Y_β}	C_{n_r}
0.04	0	-0.196	-0.101
.06	.04	-.264	-.151
.10	.12	-.400	-.249

It should be noted that a vertical tail area of 4 percent of the wing area is required to balance the unstable yawing moment of the fuselage.

In ail, nine combinations of the parameters Γ and S_f/S_w, identified as cases by the numbers 1 to 9 in the following table, mere considered:

$\frac{S_f}{S_w}$ \ Γ (deg)	0	5	10
0 04	1	4	7
.06	2	5	8
.10	3	6	9

Other factors tnat enter --,-Lo the numerical computa-tfons are

$$\mu = 12.5$$

$$K_X = 0.107$$

$$K_Z = 0.174$$

It is also necessary to determine **the** magnitude of the **ratios** C_n/C_l to be considered, Wormal ailerons, **when** so deflected as to produce *a* positive rolling moment, produce also a negative (adverse) yawing moment. The ratio of yaw-ing moment to rolling moment for plain flap, 40-percent-span ailerons (assumed for the hypothetical. airplane) *is* -0.135 (reference 6). Although larger negative values of this ratio may exist **with** certain aileron installations, the value -0,135 *has* been used **as** a representative value. It was expected that eliminating the adverse yawing moment and., possibly, applying favorable yawing moment would re-sult in more .Favorable lateral motions. The ratios 0 and 0.135 were therefore considered. For the small fin of **the** hypothetical airplane, approximately a 10° rudder deflec-tion is required aith full aileron deflection to counteract the adverse yaw of the ailerons and, in addition, to pro-duce *a* favorable yawing moment corresponding to the ratio 0.135.

Because **all** the numerical factors involved are non-dimensional, the results are directly applicable to all airplanes geometrically **similar** to tbe type assumed for the numerical computations.

RESULTS AND DISCUSSION

Lateral Motions Due to Applied Rolling Moment with Different Amounts of Accompanying Pawing Moment

The effects of changes in dihedral angle and *fin* area can be determined by comparative studies of the motions identified by the nine cases given in the preceding table. Attributes that enter such comparisons are as follows: The banking motion determines the angle of bank attainable in a given interval of time. The azimuth notion determines the magnitude and the duration of the adverse heading, and the sideslipping motion determines the angle of sideslip at any instant. It is considered desirable to attain as large a positive angle of bank as is feasible with a given positive control moment, to have the banking motion as nearly linear with time *as* possible, to prevent acy initial adverse heading and obtain *a* subsequent linear variation of heading after an interval of the order of 1 or 2 seconds, and to keep ire angle of sideslip at a minimum. All the foregoing attributes must be considered in determining any pertinent optimum condition.

A basis for analyzing the various motions having been established, consideration must be given the type of control application that initiates the motion. It is possible to consider motions initiated by rolling moment, yawing moment, or both of these moments in any combination. Sucn applications of moments are considered obtainable by deflections of the ailerons, the rudder, or both,

The curves of figure 1 represent the banking motions ϕ/C_l as defined by equation (7) for the nine combinations of parameters and the three values of the ratio C_n/C_l.

The banking motions following a deflection of the assumed ailerons, that is, motions following moments applied in the ratio $C_n/C_l = -0.135$, are shown in figure 1(a). This figure indicates that, for prolonged banking maneuvers and for each value of dihedral, increases in vertical tail area produce motions which vary more lineally with time and are of' greater magnitude. Thfs variation is due to the fact that the angle of sideslip at each instant is simultaneously reduced and therefore the magnitude of the restoring rolling moment due to sideslip io also reduced.

The effects of changes in vertical tail area become less important as the vertical tail area increases. At the start of the motion, however, cianges in vertical tail area have no appreciable effect for all tile values of dihedral considered. The 1-second timing test for aileron effectiveness is therefore of some advantage. The dihedral angle, nowever, must be taken into account if comparisons of the ailerons of different airplanes are to be made.

The curves of figure 2(a) are determined by equation (10) and are reciprocals of tne cross plots of figure 1(a) at the instants of 1, 2, 4, and 10 seconds, identified as φ_1, φ_2, φ_4, and φ_{10}. The cross plots at 1 second, identified as φ_1, show again that vertical tail area in the range covered has but little effect in an interval of the order of 1 second. After longer intervals, however, the ffect of increasing vertical tail area is pronounced and advantageous. For i st Lace, when $S_f/S_w = 0.08$, less control is needed to attain a given angle of 'Dank In 2 seconds than when $S_f/S_w = 0.04$. This effect is more pronounced for longer maneuvers and for increased values of dihedral. Because the control stick forces are directly proportional 'to the magnitude of the applied control moments, it is evident from figure 2(a) that increasing vertical tail area effects a corresponding decrease in stick forces.

Changes in vortical tail area in the range covered also have large effects on the azimuth motion, as shown by figure 3(a). (The curves of fig. 3 are determined by equation (8).) The motions for $S_f/S_w = 0.04$, for instance, are negative for the duration of the motion. An increase in S_f/S_w to 0.06 causes the motion to become positive after a relatively long interval of time. Further increases in S_f/S_w reduce this interval and decrease the magnitude of the adverse heading. This initial adverse heading cannot be totally eliminated, however, by any finite value of S_f/S_w, as is shown by figure 4(a). Figure 4(a) is a cross plot of the maximum adverse headings of figure 3,

Tne main effect of increasing vertical tail area on the sideslipping motion is to decrease the rate at which sideslip increases. This effect is evident from figure 5(a). The magnitude of the angle of sideslip may therefore be kept small, for any maneuver of the duration considered by increasing vertical tail area.

The principal effects of changes of vertical tail area may be summarized as follows: increases in area cause more linear banking motions, reduce the aileron control forces (or as a correlate, increase aileron effectiveness), decrease the magnitude and the duration of adverse heading, and retard the increase of sideslip, A consideration of all three lateral motions, ϕ, ψ, and β, thus indicates that desirable flying qualities are associated with appreciable vertical tail sizes, as long as the airplane *is* controlled predominantly by the ailerons.

The effects of changes in dihedral angle have yet to be determined. Figure 1(a) indicates that the dihedral effect opposes the banking motion resulting from aileron control. in each case. This opposing action 'is dependent upon the magnitude of the sideslip so that, for banking motions during which sideslip is not alleviated by vertical tail area, the opposition to the banking maneuver is very large. In addition, the combination of any appreciable dihedral and large angles of sideslip associated with small vertical tail areas causes the lateral oscillation to become a very large percentage of the total motion; that is, the oscillatory mode of the banking motion becomes very large with respect to the aperiodic modes. Although for such cases a consideration of the damping and the period of the lateral oscillation may indicate desirable motions, the motions are actually very erratic (for example, cases 4 and 7, fig, 1(a)).

Because the banking motion *is* diminished by the dihedral effect, it is apparent that increases in dihedral increase the aileron forces necessary to perform a given maneuver, as indicated in figure 2(a). It is also shown that, for the smaller values of vertical. tail area, the initial control moment required to perform certain maneuvers becomes very large. The indication that the necessary control application becomes infinite at certain values of S_f/S_w will be considered later.

The effects of dihedral on the banking motion having been discussed, the azimuth motion is to be considered. Figure 3(a) indicates that the azimuth is not greatly affected by dihedral, When $S_f/S_w = 0.04$ (cases 1, 4, and 7), the effect of dihedral is to prevent the heading from continuously increasing adversely. The heading, however, merely asymptotes a negative azimuth for increased values of dihedral, and this resulting motion cannot be said to be favorable. For all other cases the effect of dihedral is insignificant.

With regard to sideslipping, an inspection of figure 5(a) indicates that increasing dihedral retards the rate of increase of sideslip. This effect is large for small values of vertical tail area. It is important to note, however, that increases in dihedral prevent sideslip only at the expense of opposing the banking motion. For a given maneuver, performed by increased aileron control, the sideslip is not reduced by increases in dihedral, as indicated by figure 5(a).

On the whole, the motions as initiated by aileron control indicate that the effective dihedral should be as small as will be permitted by other criterions For cases in which the dihedral cannot be made small, increasing vertical tail area makes possible comparable motions at the expense, however, of increased control forces associated with large dihedral angles

In the foregoing discussion, only the motions initiated by conventional aileron control have been considered With such control, an adverse yawing moment is applied. The elimination of this adverse yawing moment presents a possible method of improving the resulting motions. The following discussion therefore concerns changes brought about by progressive changes in the ratio C_n/C_l from -0 135 to 0.135 Such ratios are also representative of those obtainable by simultaneous deflections of both the ailerons and the rudder

Figure 1 illustrates the improvements in the banking motions effected by increases in C_n/C_l. The motions become more nearly linear and of greater magnitude. The improvement is greatest for the smaller values of vertical tail area considered. For instance, when C_n/C_l = -0.135 (fig. 1(a)), case 4 demonstrates that it is impossible to have the airplane in a banked attitude when s = 8.25 (2.34 sec) by means of control of the type assumed. This same impossibility is demonstrated by figure 2(a), which indicates infinite control moments. Furthermore, the angle of bank after 2.34 seconds is negative although it is caused by positive control. An increase in C_n/C_l to 0 (fig. 1(b)) extends the time interval during which the bank is positive. A further increase in C_n/C_l to 0.135 entirely prevents any such erratic banking motion (case 4, fig. 1(c)). Case 7 is similarly affected.

The foregoing discussion of cases 4 and 7 (fig. 1) involves only the motions resulting from control applied instantaneously at the start of the motion and held constant thereafter. By varying correctly the ratio or magnitude of the applied control moments or both over a period of time, any maneuver can be performed. If the response to a unit control application is erratic, however, any variable control so compounded of units over a period of time as to result in any predetermined motion is correspondingly erratic. Because of the unpredictable nature of the response to control for such cases and because of the limits imposed by the pilot's skill, the ability to perform the required control manipulation for any maneuver is therefore questionable. This statement is true for the nonlinear inherent motions of cases 4 and 7 where the response to control application does not appear to be proportional to control deflections. Airplanes having such inherent lateral motions are therefore to be avoided.

It was noticed during the computations of the various motions that erratic motions occur when the oscillatory mode becomes a large percentage of the total motion, where the total motion is the summation of its periodic and aperiodic modes. The amplitude of the oscillatory mode associated with small values of S_f/S_w is very large relative to the aperiodic modes. Increases in S_f/S_w decrease the amplitude of the oscillatory mode and increase the magnitude of the aperiodic mode. This effect is large for small values of S_f/S_w and decreases as S_f/S_w increases. Simultaneously with decreases in amplitude, the period of the oscillation decreases and the damping with respect to time increases. The damping with respect to cycles, however, decreases.

Damping in terms of cycles of the lateral oscillation cannot be considered indicative of the lateral flying characteristics. Because it is the total lateral motion that demonstrates flying qualities and because the total motion is the summation of modes both periodic and aperiodic in nature, the ratio of the magnitudes of the periodic and the aperiodic modes must be kept small for favorable flying characteristics. This relation is accomplished when the period of the lateral oscillation is very short (of the order of 3 sec for the hypothetical airplane).

The effect of increasing C_n/C_l on the amount of control necessary to perform given bank maneuvers is shown in figure 2, It will be noted that, the control moments of figure 2(a) become infinite at the low values of S_f/S_w that correspond to impossible bank maneuvers, as pointed out previously. Whereas increasing C_n/C_l to 0 effects improvements in the curves, a further increase to 0.135 entirely eliminates the discontinuity of the curves in the range covered.. It should be noted that the effects of variations of vertical tail. area and dihedral are decreased by increases in C_n/C_l.

The coatrol forces being proportional to the control moments applied to tae airy lane, figure 2 also indicates a variation of control forces. The control forces are therefore decreased by increases in vertical tail area and by decreases in dihedral. Although increasing C_n/C_l indicates a decrease in applied rolling moment, the applied yawing moment, when obtained by mechanically coupling the ailerons ana tae rudder in proportion to C_n/C_l, must be considered in determining the control forces. Thus the caanges in control. forces are not; necessarily proportional to the changes In rolling moment caused by increasing C_n/C_l (fig. 2).

The effects of C_n/C_l on the azimuth notion may be determined from figure 3. Increasing C_n/C_l from -0.135 to 0.135 progressively produces better defined motions. Simultaneous application of sufficient favorable yawing moment with the rolling moment (fig. 3(c)) entirely eliminates any adverse heading.

It should also be noted that the change in heading during tae interval of tine (of the order of 2 sec) immediately following the application of control remains small for relatively large vertical tail areas in figures 3(a) and 3(b) and for all vertical. tnil areas in figure 3(c). On the other hand, tne banking motions (fig. 1) rapidly become large. Thus it is possible to perform a relatively rapid bank maneuver with the ailerons without appreciably changing the heading. Such a maneuver, for instance, is desirable for correcting: a banked attitude while landing,

Figure 4(a) is a cross plot of figure 3, showing the variation of maximum adverse heading with vertical tail area. Only one value of dihedral is considered because of the small effects of dihedral. Figure 4(a) indicates that adverse heading is entirely eliminated by applying control in the ratio $C_n/C_l > 0.129$ regardless of the value of S_f/S_w. For particular values of S_f/S_w, however, there exist particular values of C_n/C_l that prevent any adverse heading. This variation of C_n/C_l with S_f/S_w is determined by equation (11) and is shown on figure 4(b). It is indicated that these optimum values of C_n/C_l are decreased as S_f/S_w increases and as Γ becomes large.

The effect of changes in the control ratio C_n/C_l on the sideslipping motions may be determined from figure 5. Increasing C_n/C_l from -0.135 to 0.135 progressively retards the rate of increase of sideslip for all combinations of the parameters considered. It is indicated that, with small values of S_f/S_w, large angles of sideslip are likely to exist during lateral maneuvers performed by conventional aileron control.

Lateral Motions Due to Applied Yawing Moment

The effects of changes in dihedral and vertical tail area on the lateral motions resulting from applied yawing moment, such as might be obtained by rudder deflections, are indicated in figure 6. In general, it should be noted that all the lateral motions of figure 6, determined by equations (12), (13), and (14), are erratic, indicating "sloppy" maneuvering if an attempt is made to alter the flight path by means of the rudder.

Figure 6(a) indicates that increasing vertical tail area decreases the magnitude of the banking motions and thereby increases the magnitude of the applied yawing moment necessary for performing a given bank maneuver. Increasing vertical tail area also produces motions that deviate greatly from a linear variation with time. Figure 6(a) also indicates that increasing dihedral increases the magnitude of the banking motions and causes a more linear increase of bank angle with time. Figure 6(b) shows that the magnitude of the azimuth motion is decreased by increases in vertical tail area. Azimuth motions that vary irregularly with time axe also produced. Increasing dihedral (fig. 6(b)) has small effects on the azimuth motion, the character of the motion being defined essentially by vertical tail area.

Very often if is desired to change the heading without appreciably altering the angle of bar-k, as when the airplane is being alined with a runway. This maneuver is usually accomplished by means of the rudder, and the effectiveness of the rudder for this purpose may be obtained from figures 6(a) and 6(b). The rudder becomes more effective as the ratio of the azimuth motion to the banking motion becomes large. Dividing ψ/C_n by ϕ/C_n at the instant when s = 5 shows, approximately, a variation in the ratio from 3.75 to 1.75 with decreases in dihedral from 10° to 0° and, little or no variation with changes in vertical. tail area in the range covered, The same variation, although of different magnitude, exists when s = 10. It is therefore important, for this maneuver, that the dihedral angle be kept small.

The sideslip as shown by figure 6(c) is negative (that is, the airplane skids) immediately following the application of positive yawing moment and becomes positive only after an appreciable interval of tfme, that is, approximately 4 seconds or more. The effect of increasing vertical tail area is to decrease the magnitude of the skid. The sideslip during a given maneuver, however, is not appreciably altered because a correspondingly increased yawing moment must be applied in order to perform such a maneuver.

Increasing dihedral (fig. 6(c)) increases the erratic nature of the sideslipping motion and reduces its magnitude,

The motions due to applied yawing moment as a whole indicate that virtually no combination of vertical tail area and dihedral in the range covered allows well-executed lateral maneuvers with the rudder. One exception, however, is a maneuver involving intentional sideslipping, for which the rudder is an effective control, psrticularly with small dihedral. For maneuvering, then, the principal uses of the rudder are to enforce sideslip and to supply secondary assistance during aileron-controlled maneuvers.

CONCLUSIONS

The results of this theoretical analysis may be summed up in the following conclusions:

1. The motions initiated by aileron control indicated that desirable flying qualities are associated with considerable vertical tail area. Supplementary control by means of the rudder or ailerons with inherent favorable yawing moment of the order of one-eighth the rolling moment allows a reduction in vertical tail area.

2. When the airplane is controlled predominantly by the ailerons, the effective dihedral should be as small as will be permitted by other criterions. Aileron control forces may be substantially reduced by increasing vertical tail area when the directional stability is small.

3. Rudder control alone is unsatisfactory for obtaining satisfactory lateral maneuvers that involve banking and changing of course. The principal uses of the rudder are to enforce sideslip and to supplement the ailerons and, for this condition, dihedral should be kept small.

4. Damping of the lateral oscillation with respect to cycles should not be considered indicative of lateral flying characteristics.

5. For favorable lateral flying characteristics, the ratio of the magnitudes of the periodic to the aperiodic modes of the lateral motion must be kept small. This relation is apparently accomplished when the period of the lateral oscillation is very short (of the order of 3 sec for the hypothetical airplane).

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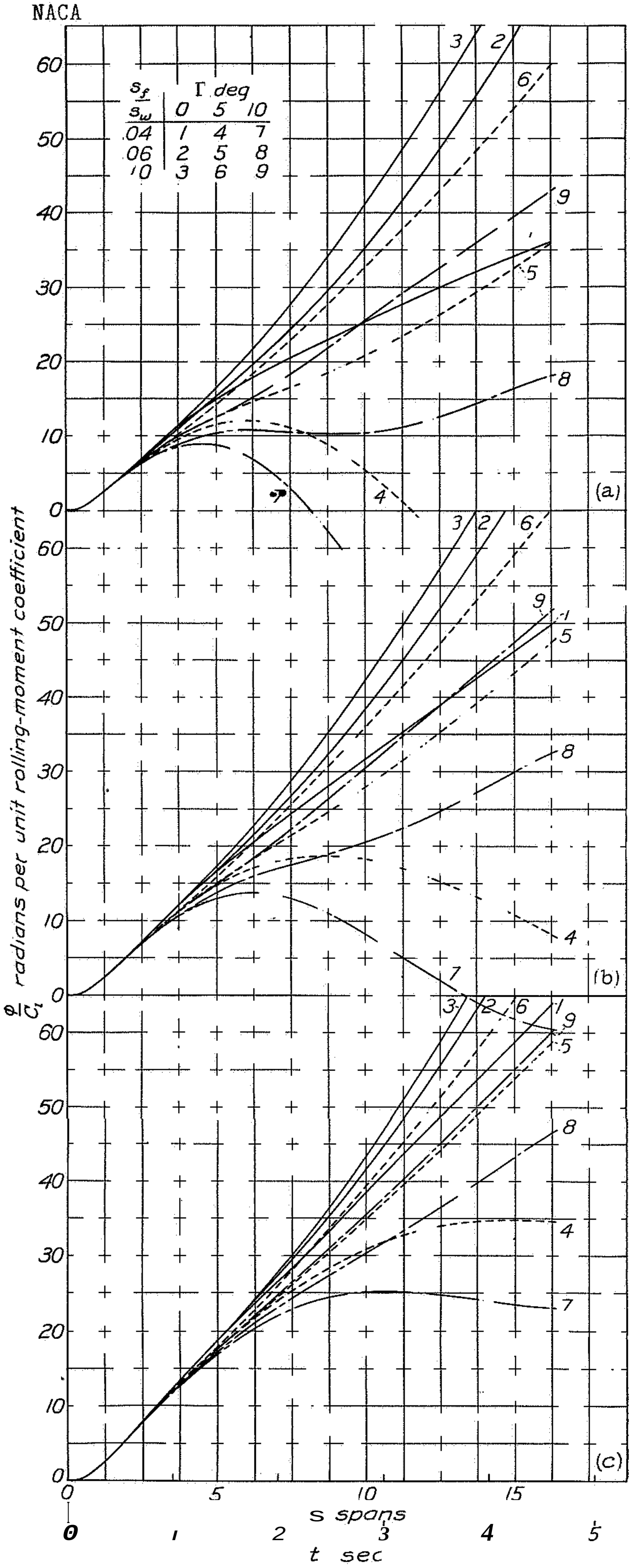
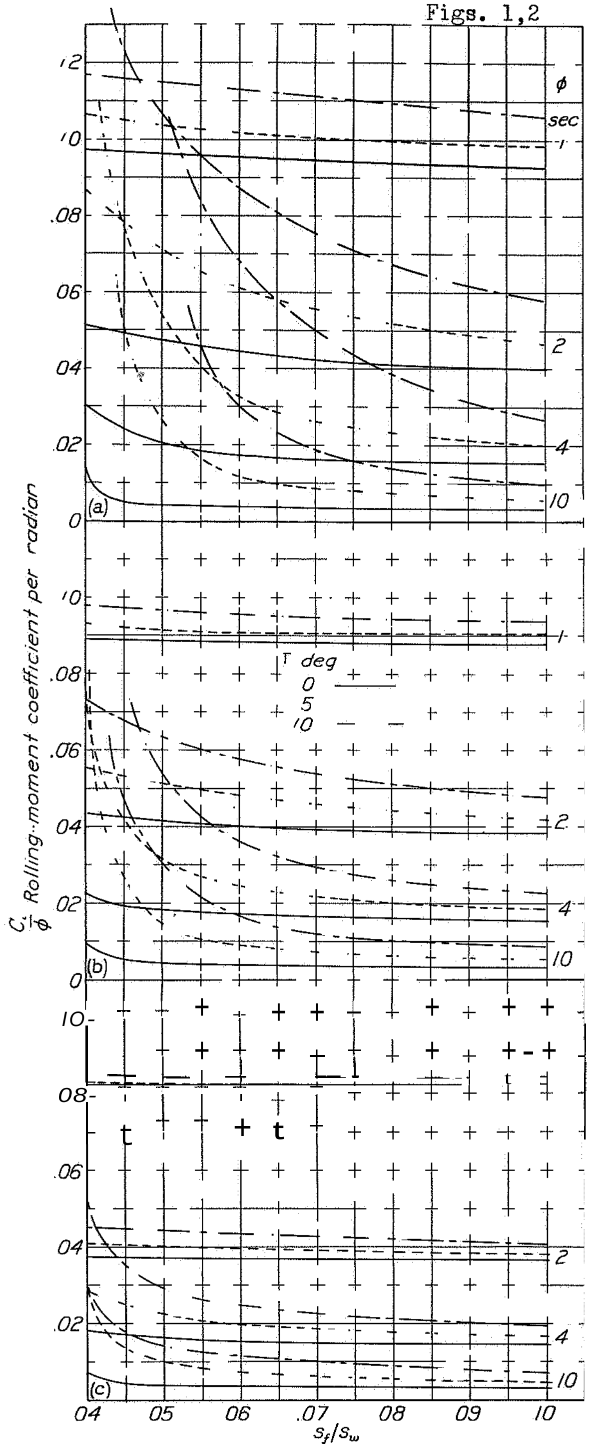


(a) $C_n/C_l = -0.135$ (b) $C_n/C_l = 0$ (c) $C_n/C_l = 0.135$

Figure 1.- The banking motion due to a unit rolling moment as influenced by changes in vertical tail area, dihedral, and control characteristics.



(a) $C_n/C_l = -0.135$. (b) $C_n/C_l = 0$ (c) $C_n/C_l = 0.135$.

Figure 2.- The magnitude of control moments necessary to perform certain banking maneuvers as influenced by changes in vertical tail area, dihedral, and control characteristics.

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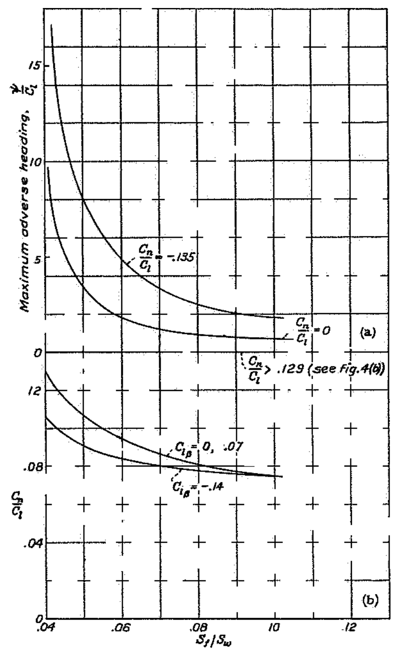
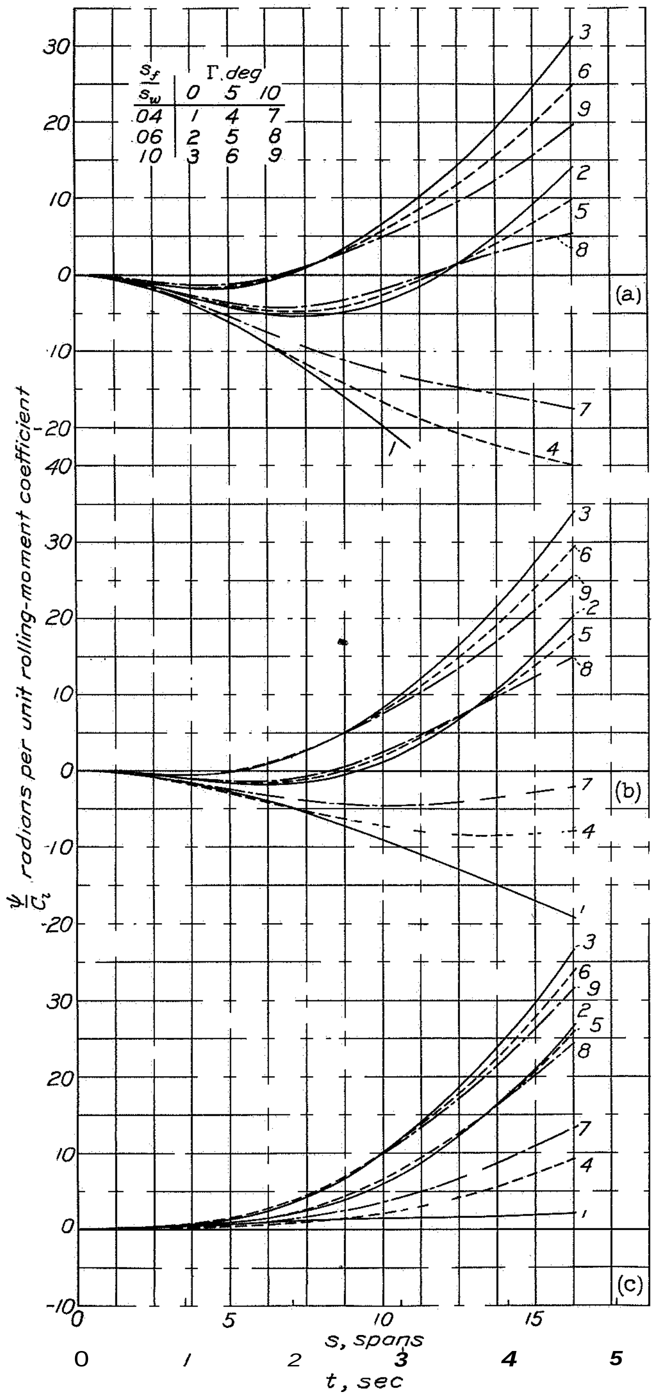


(a) $\Gamma = 5^{\circ}$. (b) $\psi/C_l = 0$.

Figure 4.- The adverse heading as influenced by changes in vertical tall area, dihedral, and control characteristics.



(a) $C_n/C_l = -0.135$. (b) $C_n/C_l = 0$. (c) $C_n/C_l = 0.135$.

Figure 3.- The azimuth motion due to a unit rolling moment as influenced by changes in vertical tail area, dihedral, and control characteristics.

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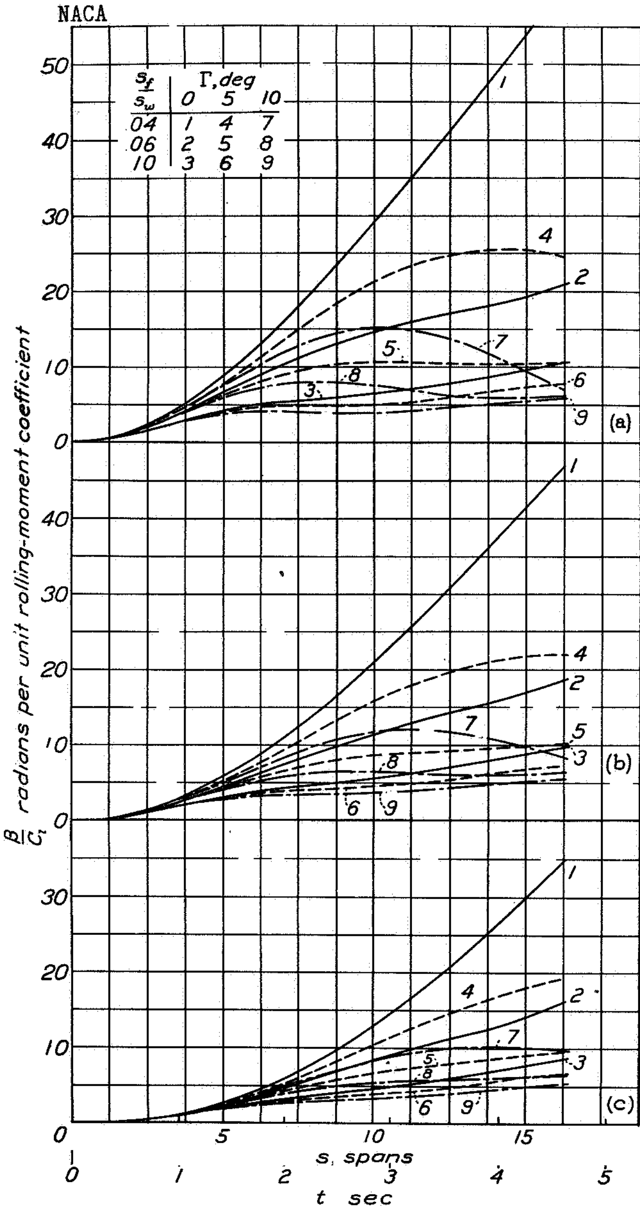
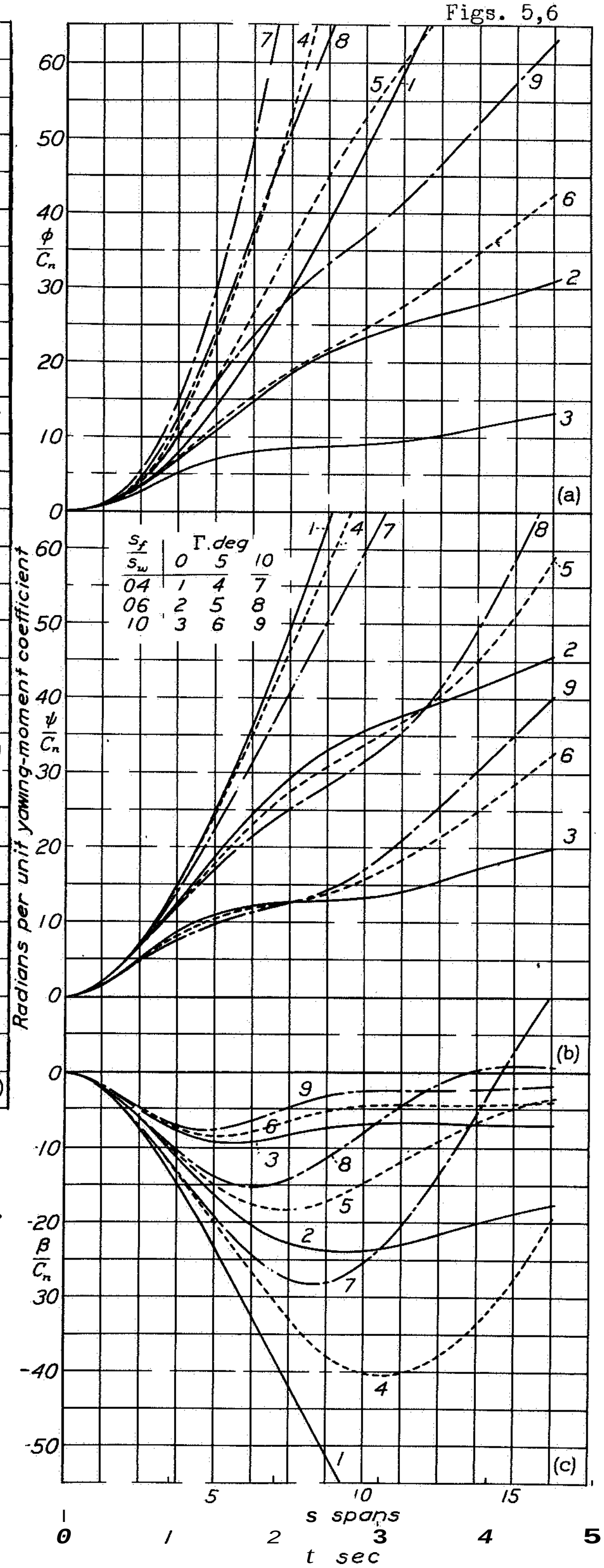


(a) C_n/C_l = -0.135. (b) C_n/C_l = 0.
(c) C_n/C_l = 0.135.

Figure 5.- The sideslipping motion due to a unit rolling moment as influenced by changes in vertical tail area, dihedral, and control characteristics.



(a) Rolling motion (b) Azimuth motion.
(c) Sideslipping motion.

Figure 6 The lateral motions due to a unit yawing moment as influenced by changes in vertical tail area and dihedral